Introducing Radar System Structure to Dropped Channel Polarimetric SAR

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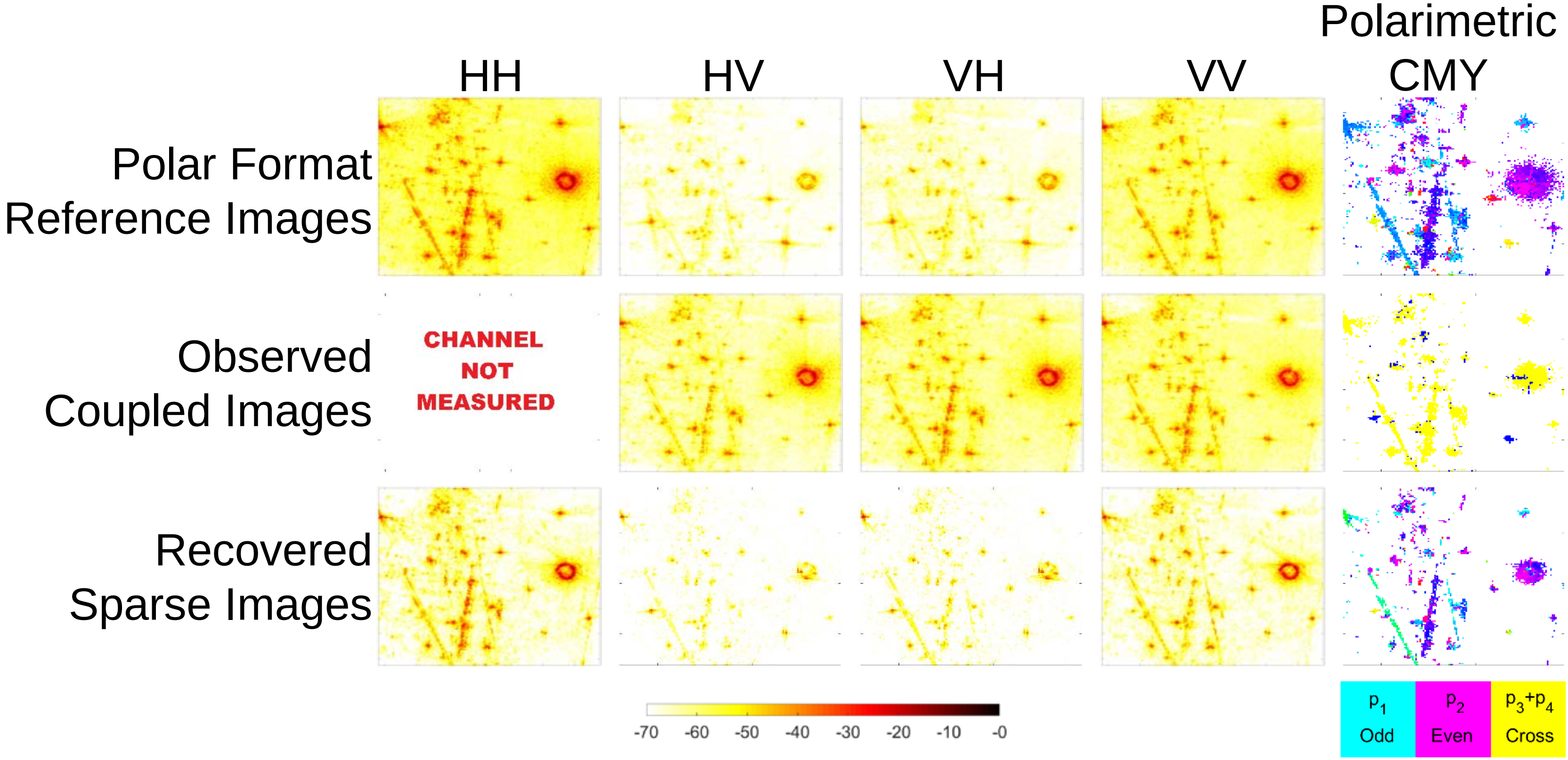
Overview

- PolSAR bistatic system model and CS framework
- Measurement and Dictionary Matrices
  - Antenna Coupling
  - Channel Selection
  - Polarimetric canonical scatterer representation
- Results – Recovery of Four Polarimetric Channels
  - Active and Passive antenna Coupling
  - One or Two Dropped Channels

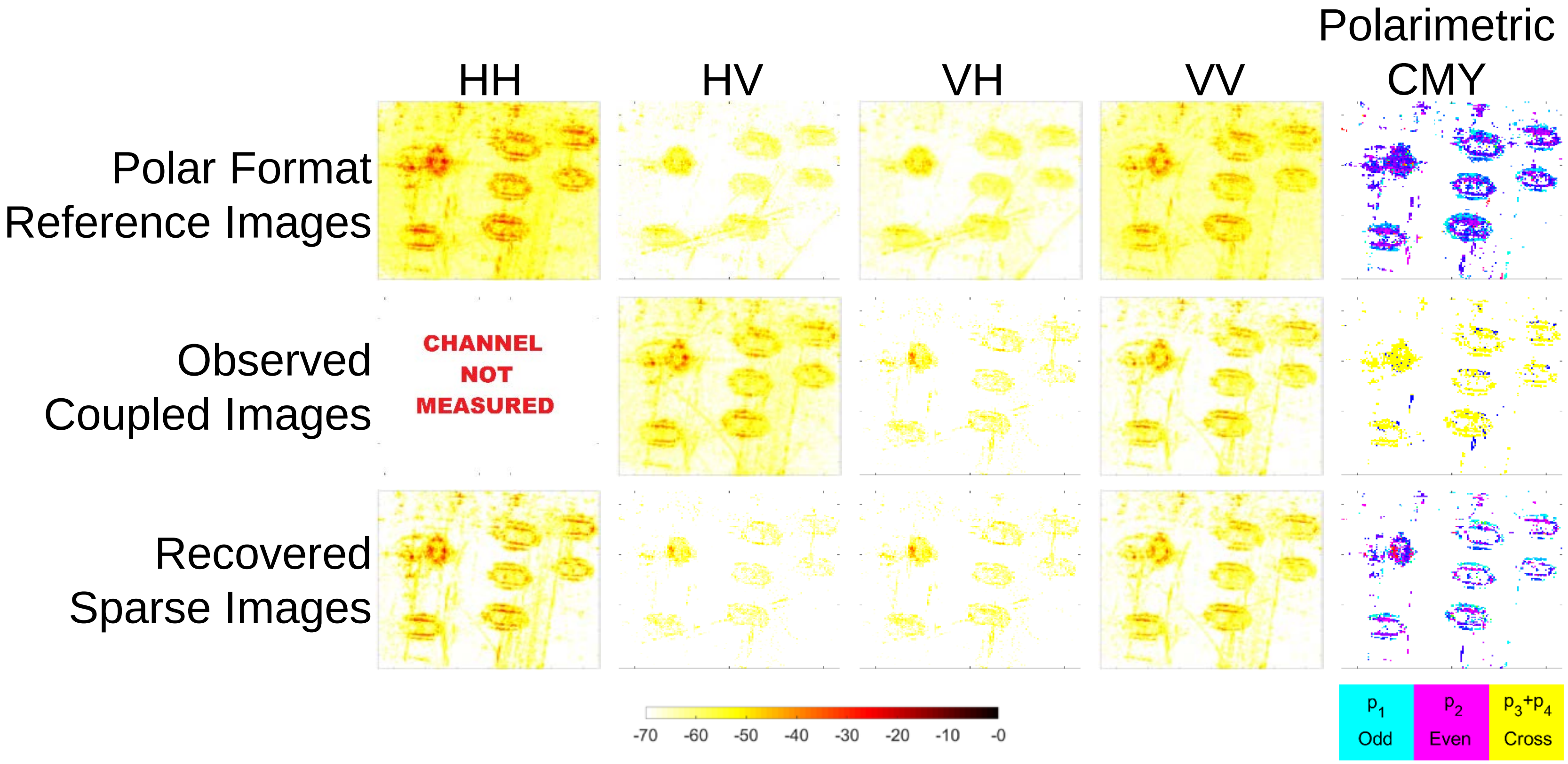
What is Dropped-Channel PolSAR?

- Jackson and Lee-Elkin first introduced Dropped-Channel PolSAR at IEEE Radar Conf 2017
- Uses Channel Coupling and CS framework to compress along polarization dimension

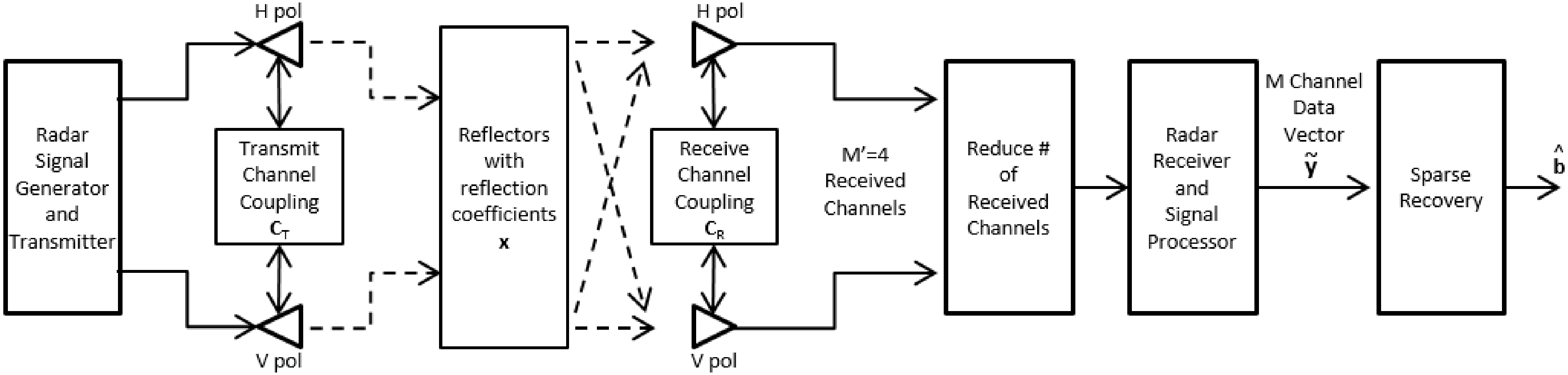


Parking Lot Example

- Single and Double-bounce scattering off vehicles is recovered with Basis Pursuit Denoising with the HH channel omitted.

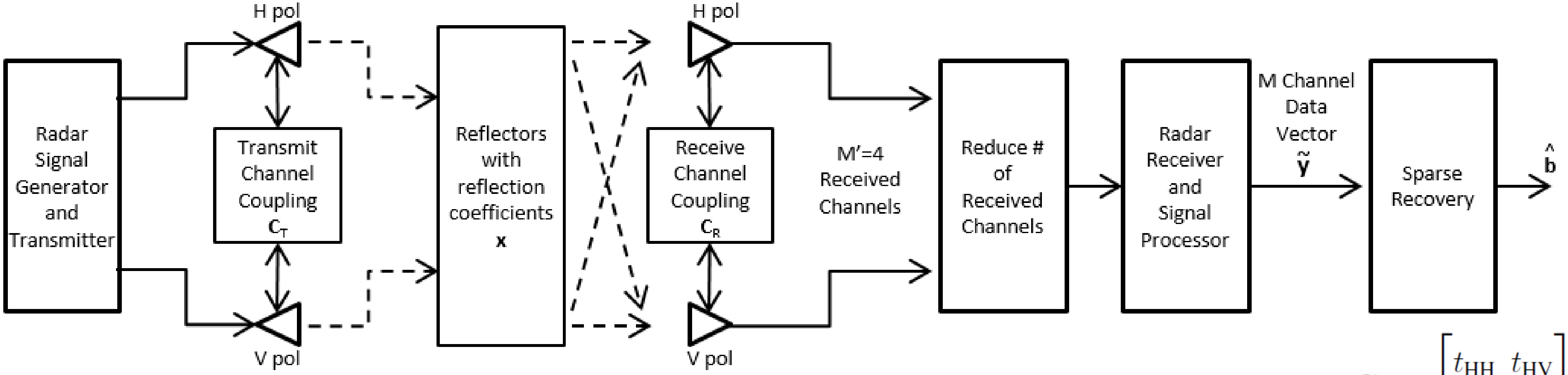


PolSAR System Model



- Channel coupling (crosstalk) occurs between transmit antennas and between receive antennas
- Coupling provides linear mixing required for CS framework
- Transmitted electric field polarization may be scaled and/or rotated by reflectors in the scene
  - Both H and V receive antennas may collect scaled versions of transmitted H or V pol signal
  - 16 possible paths through Tx-reflector-Rx path

PolSAR System Model



$$C_T = \begin{bmatrix} t_{\mathrm{HH}} & t_{\mathrm{HV}} \\ t_{\mathrm{VH}} & t_{\mathrm{VV}} \end{bmatrix}$$

$$C_R = \begin{bmatrix} r_{\mathrm{HH}} & r_{\mathrm{HV}} \\ r_{\mathrm{VH}} & r_{\mathrm{VV}} \end{bmatrix}$$

- Channel coupling on transmit and receive modeled by
- Overall system coupling: $C = C_T \otimes C_R^T$
- Channel coupling can be estimated via radar calibration
- Traditional processing: remove coupling by applying **C**$^{-1}$ to received channel data and process all channels
- Proposed processing: reduce # channels and do sparse recovery to recover channel(s) and remove coupling

Mathematical PolSAR System Model

- Recover sparse coefficients **b** from reduced channel measurements $\tilde{\boldsymbol{y}}$

$$\min_{\mathrm{b}} \|\mathrm{b}\|_1 \quad \text{s.t.} \quad \|\tilde{\mathrm{y}} - \tilde{\boldsymbol{A}}_M (\boldsymbol{J}\boldsymbol{C} \otimes \boldsymbol{I}_{N'})(\boldsymbol{P} \otimes \boldsymbol{S})\, \mathrm{b}\|_2 \le \epsilon$$

Measured Image Vector

Measurement Matrix

Dictionary

$$\begin{bmatrix} y_{HV}\\ y_{VH}\\ y_{VV}\end{bmatrix} = \left(\begin{bmatrix} \boldsymbol{A}_1 & & \\ & \ddots & \\ & & \boldsymbol{A}_M\end{bmatrix}(\boldsymbol{J}\boldsymbol{C}\otimes \boldsymbol{I}_{N'})\right)\left(\begin{bmatrix}x_{HH}\\x_{HV}\\x_{VH}\\x_{VV}\end{bmatrix}+\begin{bmatrix}w_{HH}\\w_{HV}\\w_{VH}\\w_{VV}\end{bmatrix}\right)$$

$$\underbrace{\tilde{\boldsymbol{y}}}_{\text{Measured Image Vector}} = \underbrace{\tilde{\boldsymbol{A}}}_{\text{Measurement Matrix}}(\underbrace{\boldsymbol{x}}_{\text{Scene Reflectivity Vector}} + \underbrace{\boldsymbol{w}}_{\text{Clutter Vector}})$$

- $\mathbf{A}_m$ = m^th^ channel point spread convolution matrix
- **J** = Channel selection matrix (keep M of M' channels)
- **C** = M'xM' Channel coupling: $\mathbf{C}=\mathbf{C}_T\otimes\mathbf{C}_R^T$
- M'N' unknowns in **x**; recover from MN measurements

$$\boldsymbol{x} = (\boldsymbol{P} \otimes \boldsymbol{S})\boldsymbol{b}$$

$$\boldsymbol{P} = \frac{1}{\sqrt{2}}\begin{array}{cccc} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & -j \\ 0 & 0 & 1 & j \\ 1 & -1 & 0 & 0 \\ \text{p}_1 \text{ odd} & \text{p}_2 \text{ even} & \multicolumn{2}{c}{\text{p}_3\text{+p}_4 \text{ cross}} \end{array}$$

- **P** = Polarization dictionary
  - Pauli basis models canonicals
- **S** = Spatial dictionary
  - Isotropic point model $\mathbf{S}=\mathbf{I}_{N'}$
- **b** = Sparse coefficients

Radar and Recovery Design Considerations

- Measurement Matrix:
  - Point spread function determined by radar flight path, aperture extent, bandwidth, and operating frequency
  - Coupling matrix structure dictated by physics

$$C = C_T \otimes C_R^T$$

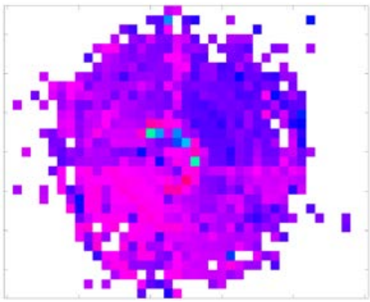
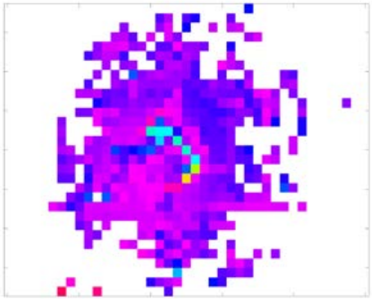
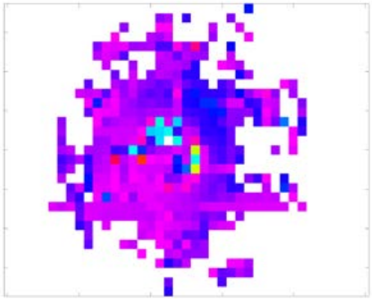
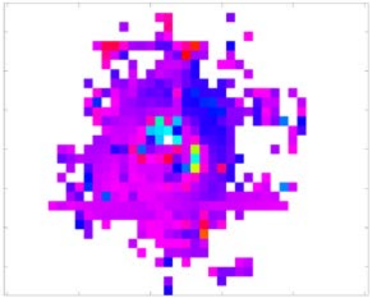
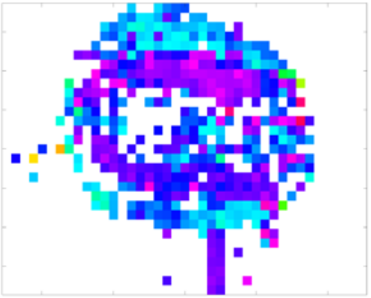
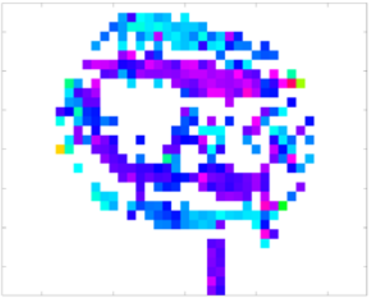
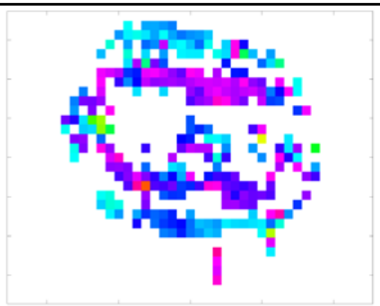
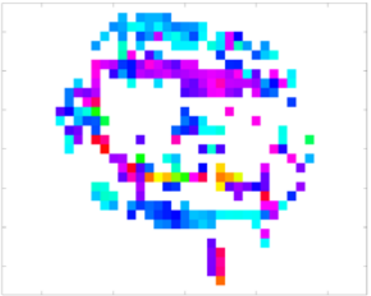
$$C_T = \begin{bmatrix} t_{\mathrm{HH}} & t_{\mathrm{HV}} \\ t_{\mathrm{VH}} & t_{\mathrm{VV}} \end{bmatrix}$$

$$C_R = \begin{bmatrix} r_{\mathrm{HH}} & r_{\mathrm{HV}} \\ r_{\mathrm{VH}} & r_{\mathrm{VV}} \end{bmatrix}$$

    - Reciprocity requires symmetric $\mathbf{C}_T$, $\mathbf{C}_R$
    - Active components may break reciprocity

  - Channel selection **J** should be designed in conjunction with **C**
    - **J** may drop channels
    - **J** may further combine channels (e.g. monostatic radar)

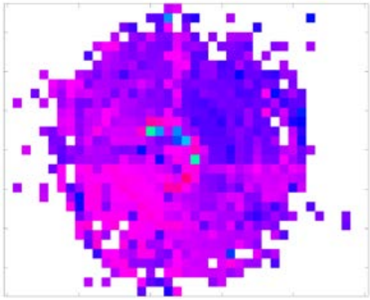
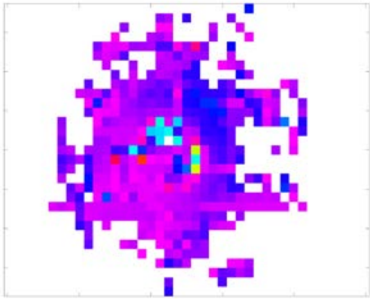
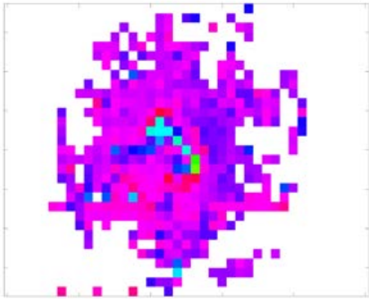
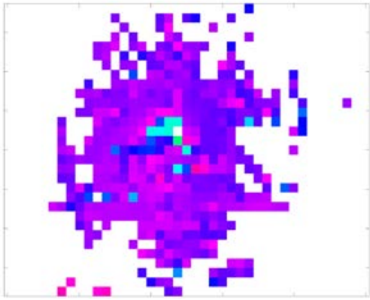
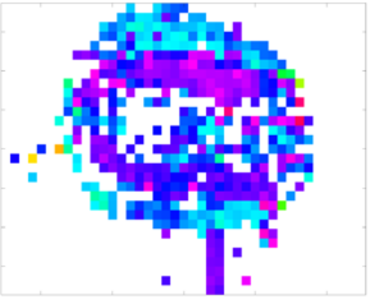
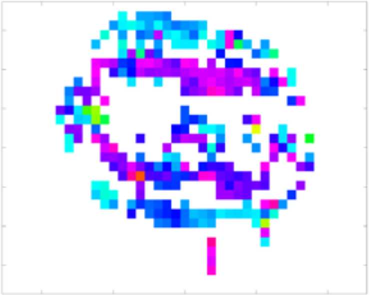
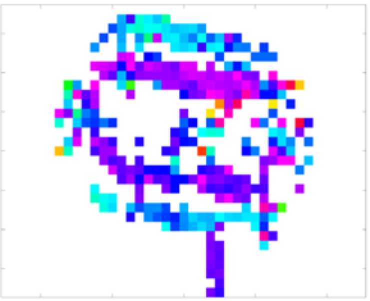
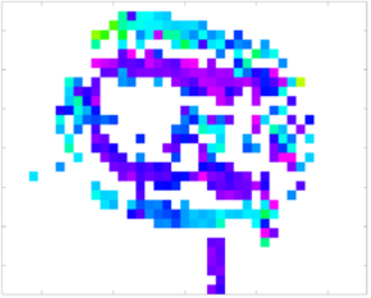
Vary Coupling Matrix C;
Drop HH Channel

	Reference	**Random C**	**Active Coupling**	**Passive Coupling**
	p_1 Odd p_2 Even p_3+p_4 Cross	$C=\begin{bmatrix} 1.00 & 0.40^{\angle -2.86} & 0.33^{\angle 1.91} & 0.88^{\angle -1.75} \\ 0.97^{\angle 2.15} & 1.00 & 0.15^{\angle 0.29} & 0.09^{\angle -0.77} \\ 0.65^{\angle -1.66} & 0.27^{\angle -0.67} & 1.00 & 0.93^{\angle -2.62} \\ 0.23^{\angle -1.29} & 0.26^{\angle 0.34} & 0.12^{\angle -1.76} & 1.00 \end{bmatrix}$	$\boldsymbol{C}=\frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1^{\angle -1.64} \\ 1^{\angle -1.52} & 1 \end{bmatrix} \otimes 0.86\begin{bmatrix} 1 & 0.6^{\angle -2.79} \\ 0.6^{\angle -0.45} & 1 \end{bmatrix}$	$\boldsymbol{C}=\frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1^{\angle -1.64} \\ 1^{\angle -1.64} & 1 \end{bmatrix} \otimes 0.86\begin{bmatrix} 1 & 0.6^{\angle -0.45} \\ 0.6^{\angle -0.45} & 1 \end{bmatrix}$
top-hat				
vehicle				

- Sparse image recovered with proper polarization response due to channel recovery

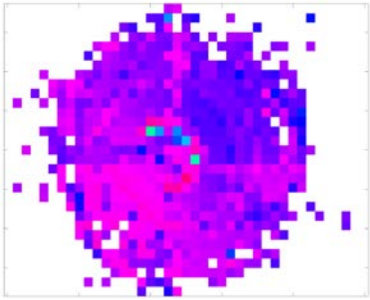
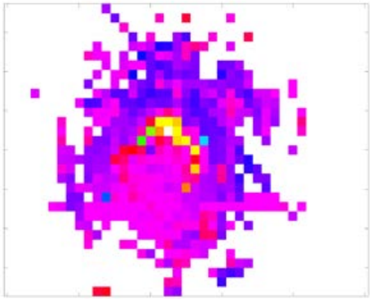
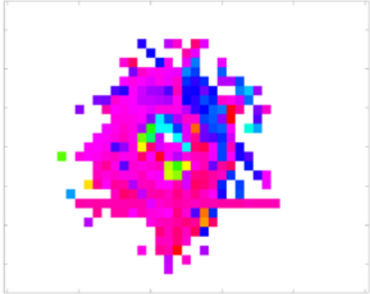
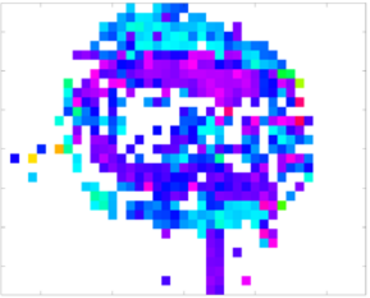
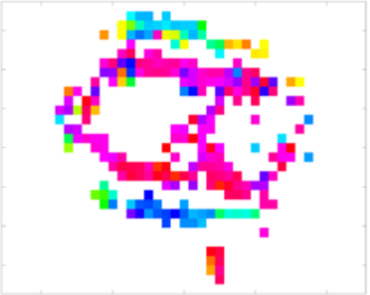
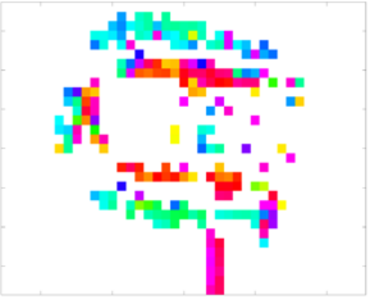
Active Coupling Matrix C; Drop Various Channels

$$C = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1^{\underline{\angle -1.64}} \\ 1^{\underline{\angle -1.52}} & 1 \end{bmatrix} \otimes 0.86 \begin{bmatrix} 1 & 0.6^{\underline{\angle -2.79}} \\ 0.6^{\underline{\angle -0.45}} & 1 \end{bmatrix}$$

	Reference	**Drop HH**	**Drop HV**	**Drop VV**
	p_1 Odd p_2 Even p_3+p_4 Cross	$\mathbf{J} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$	$\mathbf{J} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$	$\mathbf{J} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$
top-hat				
vehicle				

- Sparse image recovery works if drop co-pol or cross-pol channel

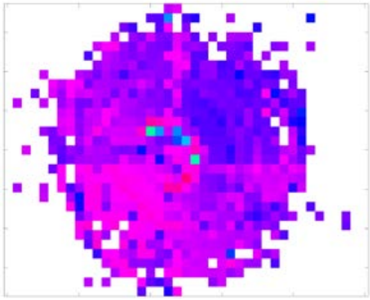
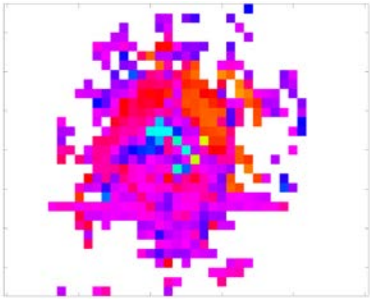
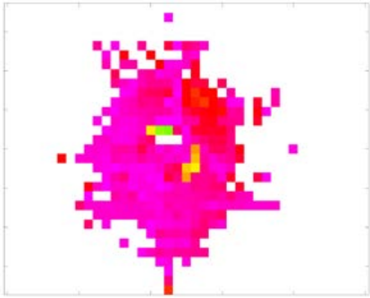
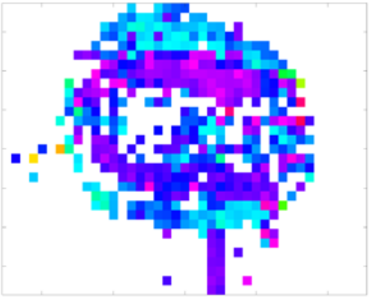
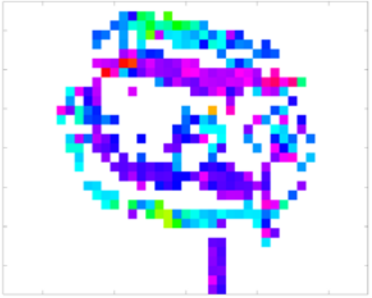
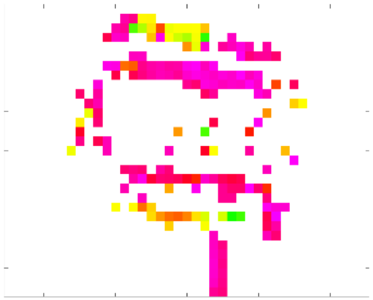
Active Coupling Matrix C; Drop Two Channels

	Reference	**Drop HH and VV**	**Drop HH and VH**
	p_1 Odd p_2 Even p_3+p_4 Cross	$\mathbf{J}=\begin{bmatrix}0&1&0&0\\0&0&1&0\end{bmatrix}$	$\mathbf{J}=\begin{bmatrix}0&1&0&0\\0&0&0&1\end{bmatrix}$
top-hat			
vehicle			

$$\mathbf{C}=0.78\begin{bmatrix}1 & 0.8^{\angle 3.03}\\ 0.91^{\angle 0.29} & 0.91\end{bmatrix}\otimes 0.93\begin{bmatrix}1 & 0.4^{\angle -1.63}\\ 0.4^{\angle -1.63} & 1\end{bmatrix}$$

- Sparse image recovery with 2 of 4 channels dropped will improve as optimize $\tilde{A}$**(J,C)**

Active Coupling Matrix C; Monostatic Average Two Channels

	Reference	**Monostatic Avg. HV and VH**	**Monostatic Avg. HV and VH; Drop HH**
	p_1 Odd p_2 Even p_3+p_4 Cross	$\mathbf{J}=\begin{bmatrix}1&0&0&0\\0&0.5&0.5&0\\0&0&0&1\end{bmatrix}$	$\mathbf{J}=\begin{bmatrix}0&0.5&0.5&0\\0&0&0&1\end{bmatrix}$
top-hat			
vehicle			

$$\mathbf{C}=0.78\begin{bmatrix}1 & 0.8^{\angle 3.03}\\ 0.91^{\angle 0.29} & 0.91\end{bmatrix}\otimes 0.93\begin{bmatrix}1 & 0.4^{\angle -1.63}\\ 0.4^{\angle -1.63} & 1\end{bmatrix}$$

- Sparse image recovery with channel-averaging/dropping will improve as optimize $\widetilde{A}$**(J,C)**

Conclusion

- Sparse recovery of all channels possible for various cases
    - Drop co-pol or cross-pol
    - Passive or active coupling in Tx and Rx
- Results will improve as optimize measurement matrix $\tilde{\boldsymbol{A}}$
    - Coupling design
    - Channel selection
    - Flight path (point spread)
- Future work will also consider efficient recovery based on structure of $\tilde{\boldsymbol{A}}$

Back-up Slides

Recovery vs. Signal-to-Clutter Ratio

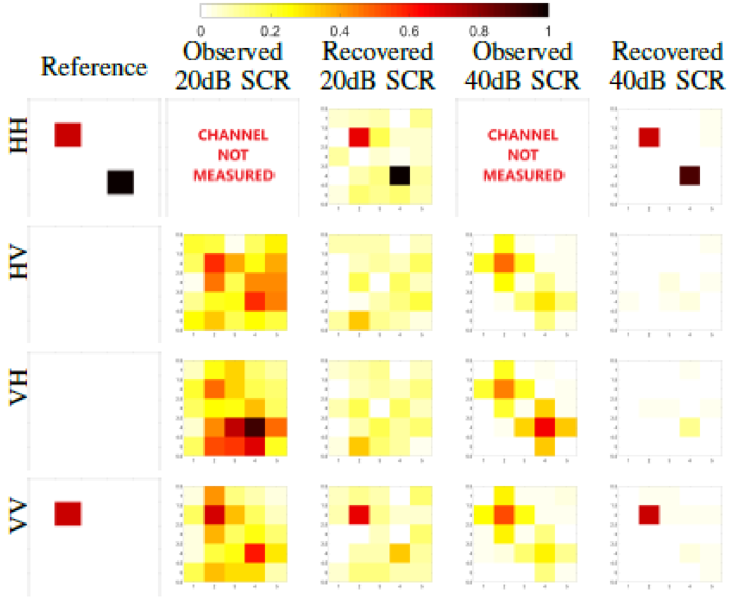


Fig. 2. Reference, Observed, and Recovered images of point targets when the HH channel is not observed and a Pauli basis is used for recovery.

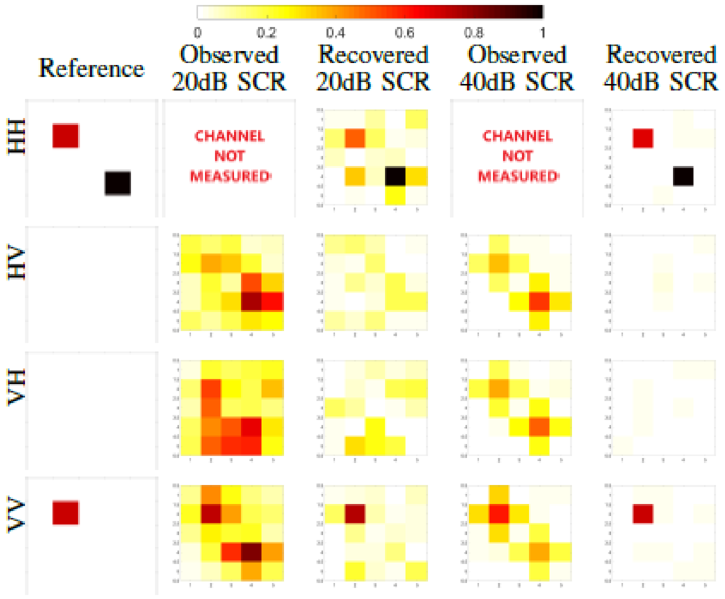


Fig. 3. Reference, Observed, and Recovered images of point targets when the HH channel is not observed and a Cardinal basis $P = I$ is used for recovery.

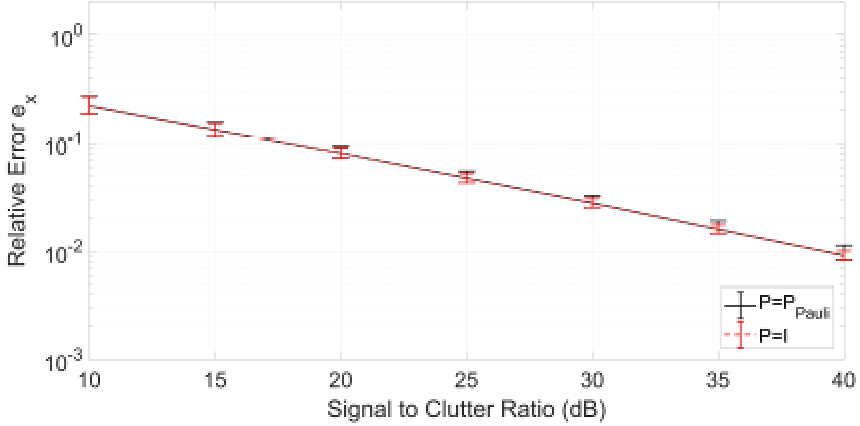


Fig. 6. Relative Error versus SCR for $K/(QN') = 0.05$. HH channel dropped.

Relative Error vs. Sparsity

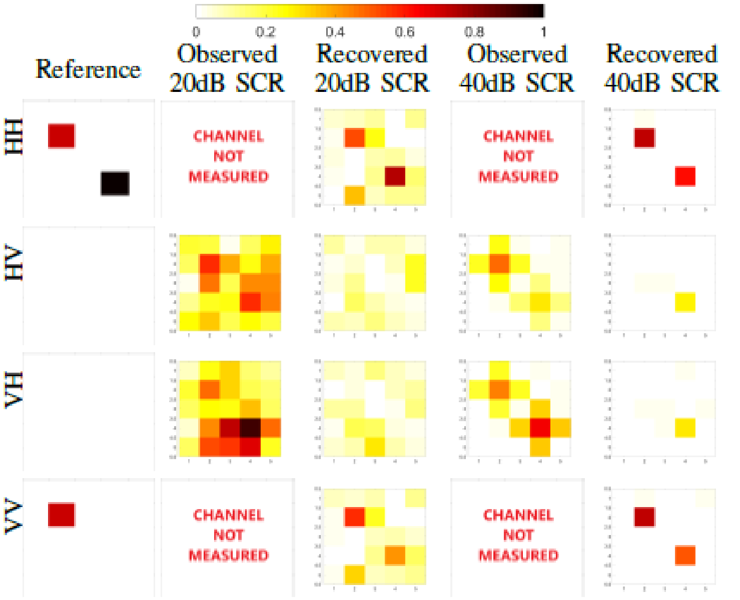


Fig. 4. Reference, Observed, and Recovered images of point targets when neither the HH nor VV channels are observed and a Pauli basis is used for recovery.

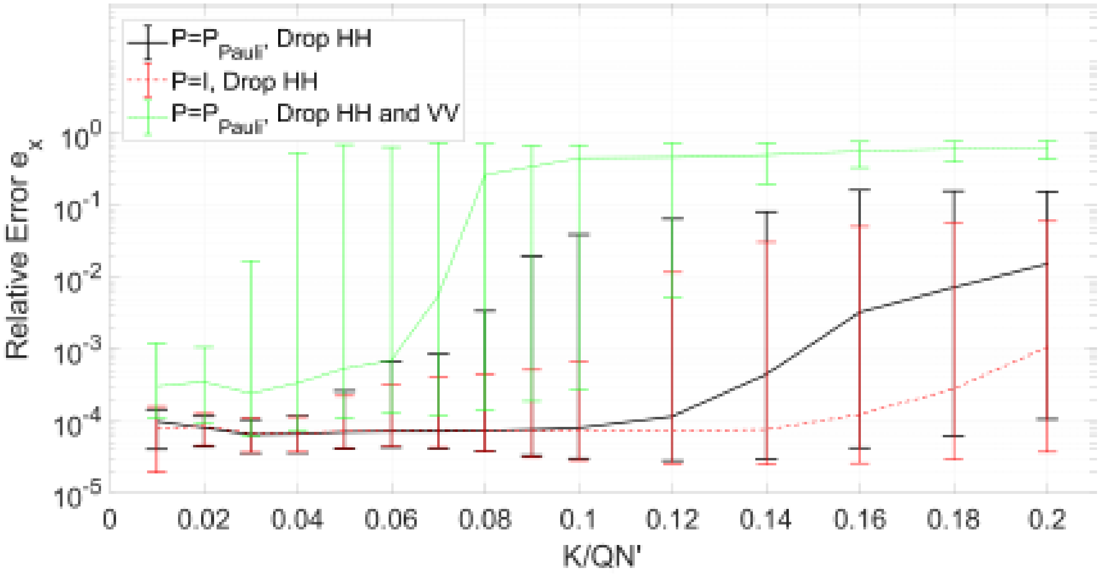


Fig. 5. Relative Error versus sparsity: SCR$= \infty$